Name: Solutions

This assignment is **optional**. If you complete this assignment, the score you earn will replace your current lowest homework grade. There is **no late deadline** for this assignment and so it **must** be handed in on **Wednesday**. This assignment consists of nine questions, for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help.

1. For each of the following, find a possible formula for the function represented by the data.

(a)

+4 +4 +4

x	0	4	8	12
$f(x)$	−3	6	15	24

+9 +9 +9

linear

$$y - y_1 = m(x - x_1)$$

$$f(x) - (-3) = \frac{9}{4}(x - 0)$$

$$m = \frac{15-6}{8-4} = \frac{9}{4} = 2.25$$

Answer: $f(x) = \frac{9}{4}x - 3$

(b)

+1 +1 +1

t	0	1	2	3
$p(t)$	14.148	18.864	25.152	33.536

+4.716 +8.384

Not linear

P_0

$$\frac{18.864}{14.148} = \frac{25.152}{18.864} = \frac{33.536}{25.152} = \frac{4}{3} = a$$

Answer: $p(t) = 14.148\left(\frac{4}{3}\right)^t$

(c)

+1 +1 +1

s	1	2	3	4
$q(s)$	12.005	8.575	6.125	4.375

−3.430 −1.750

Not linear

Not P_0

$$\frac{8.575}{12.005} = \frac{6.125}{8.575} = \frac{4.375}{6.125} = \frac{5}{7} = a$$

$$q(1) = P_0\left(\frac{5}{7}\right)^1 = 12.005$$

$$\Rightarrow P_0 = \frac{7}{5} \times 12.005$$

$$= 16.807$$

Answer: $q(s) = 16.807\left(\frac{5}{7}\right)^s$

2. A company releases a new phone in 2007. The value of the phone decreases linearly per year. In ~~2009~~ 2 the phone was selling for $757. In ~~2013~~ 6 the phone was selling for $571.

   (a) Based on this change in price, find a function that represents the price of the phone as a function of t years since 2007.

   $m = \frac{571-757}{6-2}$

   $= -46.50$

   $y - y_1 = m(x - x_1)$

   $C(t) - 757 = -46.5(t-2)$

   $C(t) = -46.5t + 90 + 757$

   Answer: $C(t) = -46.5t + 847$

   (b) How much did the phone cost when it was released?

   Answer: $847

   (c) In 2010, the cost, in dollars, to build q phones was modelled by the equation $c(q) = 227q + 8,162$. Find an equation that models the profit function for this company in 2010.

   Price in 2010 $= C(3) = 710.50$

   $R(q) = 710.5q$

   $\pi(q) = R(q) - C(q)$

   $= 710.5q - (227q + 8162)$

   Answer: $\pi(q) = 483.5q - 8{,}162$

3. Carbon-14 has a half life of 5,730 years. In 2016, a wooden sculpture contained 92.7% of its carbon-14.

   $a = \frac{1}{2}$

   (a) Find a formula, $C(t)$, that gives the percentage of carbon-14 expected to be present in the sculpture t years after it was painted.

   1 year $= \frac{1}{5730} \times 5730$ years

   loses $\frac{1}{2}^{\frac{1}{5730}}$ loses $1/2$

   Answer: $C(t) = (1/2)^{\frac{t}{5730}}$

   (b) Use this formula to estimate the year that the sculpture was created.

   $\left(\frac{1}{2}\right)^{t/5730} = 0.927$

   $\Rightarrow \frac{t}{5730} \ln(1/2) = \ln(0.927)$

   $t = 5730 \frac{\ln(0.927)}{\ln(1/2)} = 626.63$

   $2016 - 626.63 = 1389.37$

   Answer: ≈ 1389

Cont.

4. A certain bacteria was introduced into a system in 1992. In 1992 the population was 3,125. In 1997 the population was 7,776.

   (a) Assuming exponential growth, find the (continuous) rate of growth of the bacteria population between 1992 and 1997.

   $P(t) = P_0 e^{kt} = 3125 e^{kt}$

   $P(5) = 3125 e^{5k} = 7776$

   $\Rightarrow e^{5k} = \frac{7776}{3125}$

   $\Rightarrow 5k = \ln\left(\frac{7776}{3125}\right)$

   $\Rightarrow k = \frac{1}{5}\ln\left(\frac{7776}{3125}\right)$

   Answer: $k = \frac{1}{5}\ln\left(\frac{7776}{3125}\right) \approx 0.1823$

   (b) Find a formula, $P(t)$, for the population as a function of the number of years, t, since 1992.

   Answer: $P(t) = 3125\, e^{\frac{1}{5}\ln\left(\frac{7776}{3125}\right)t}$

   (c) Estimate the population of the bacteria in the year ~~2001~~ 9.

   Answer: $P(9) \approx 16124$

5. (a) Let $13 = 6e^{3t}$. Solve for t using *natural* logarithms.

   $\Rightarrow \frac{13}{6} = e^{3t}$

   $\Rightarrow \ln\left(\frac{13}{6}\right) = 3t$

   Answer: $t = \frac{1}{3}\ln\left(\frac{13}{6}\right)$

   (b) Let $7e^{5t} = 9e^{12t}$. Solve for t using *natural* logarithms.

   $\Rightarrow \frac{7}{9}e^{5t} = e^{12t}$

   $\Rightarrow \frac{7}{9} = \frac{e^{12t}}{e^{5t}} = e^{7t}$

   $\Rightarrow \ln(7/9) = 7t$

   Answer: $t = \frac{1}{7}\ln(7/9)$

Cont.

6. A function $f(x)$ crosses the points $(1, 2625)$ and $(4, 7203)$. Find a formula for $f(x)$ if

(a) $f(x)$ is a *linear* function.

$$m = \frac{7203 - 2625}{4-1} = 1526$$

$$y - y_1 = m(x - x_1)$$

$$f(x) - 2625 = 1526(x-1)$$

Answer: $f(x) = 1526x + 1099.$

(b) $f(x)$ is an *exponential* function.

$$f(x) = P_0 a^t$$

$$\frac{f(4)}{f(1)} = \frac{\cancel{P_0} a^4}{\cancel{P_0} a^1} = a^3 = \frac{7203}{2625} = 2.744$$

$$\Rightarrow a = \sqrt[3]{2.744} = 1.4$$

$$f(1) = P_0 (1.4)^1 = 2625$$

$$\Rightarrow P_0 = \frac{2625}{1.4} = 1875$$

Answer: $f(x) = 1875(1.4)^x$

7. Fred invests \$1,000 into an account that pays 4.38% interest per year. Find an expression, $A(t)$, for the amount in the account after t years if the interest is compounded;

(a) Monthly

Answer: $A(t) = 1000\left(1 + \frac{0.0438}{12}\right)^{12t}$

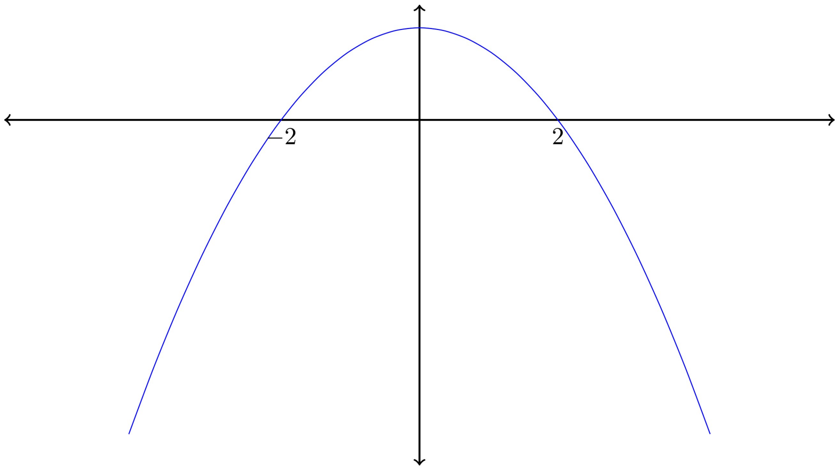
(b) Daily

Answer: $A(t) = 1000\left(1 + \frac{0.0438}{365}\right)^{365t}$

(c) Continuously

Answer: $A(t) = 1000e^{0.0438t}$

8. A graph of $f'(x)$ is given below.



(a) At what intervals is the function $f(x)$ increasing?

Answer: $-2 < x < 2$

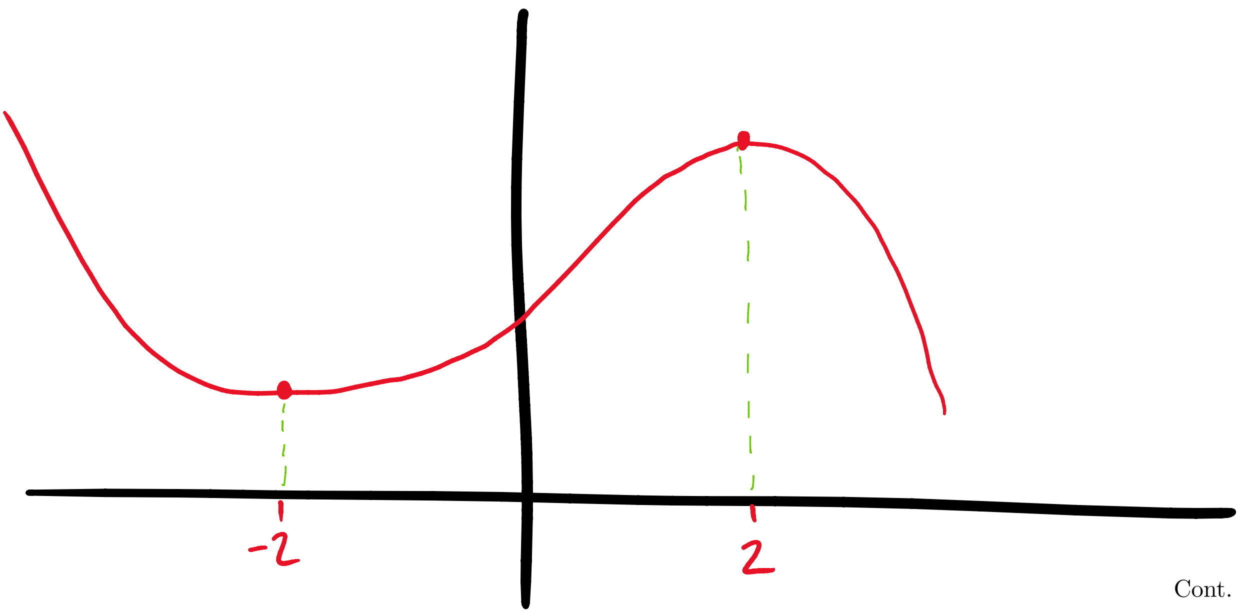
(b) At what intervals is the function $f(x)$ decreasing?

Answer: $x < -2$ and $2 < x$

(c) At what intervals is the function $f(x)$ stationary?

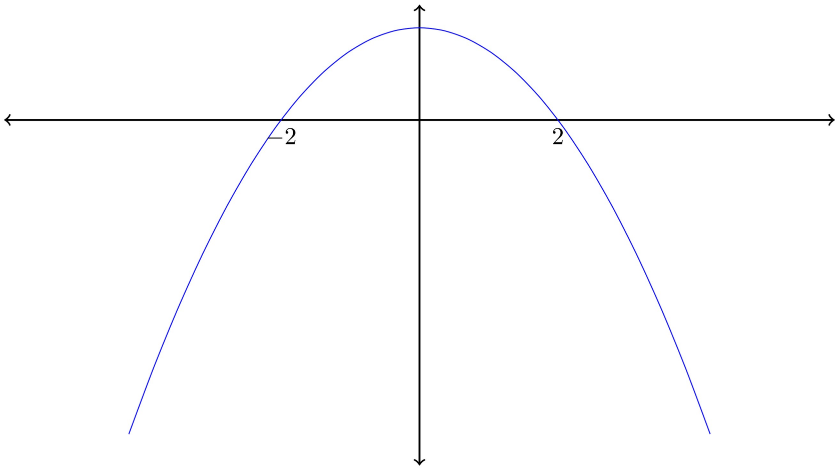
Answer: $x = -2, 2$

(d) Sketch a possible candidate for the function $f(x)$. You need only label the x-axis.



Cont.

9. A graph of $f(x)$ is given below.



(a) At what intervals is the function $f'(x) > 0$?

Answer: $x < 0$

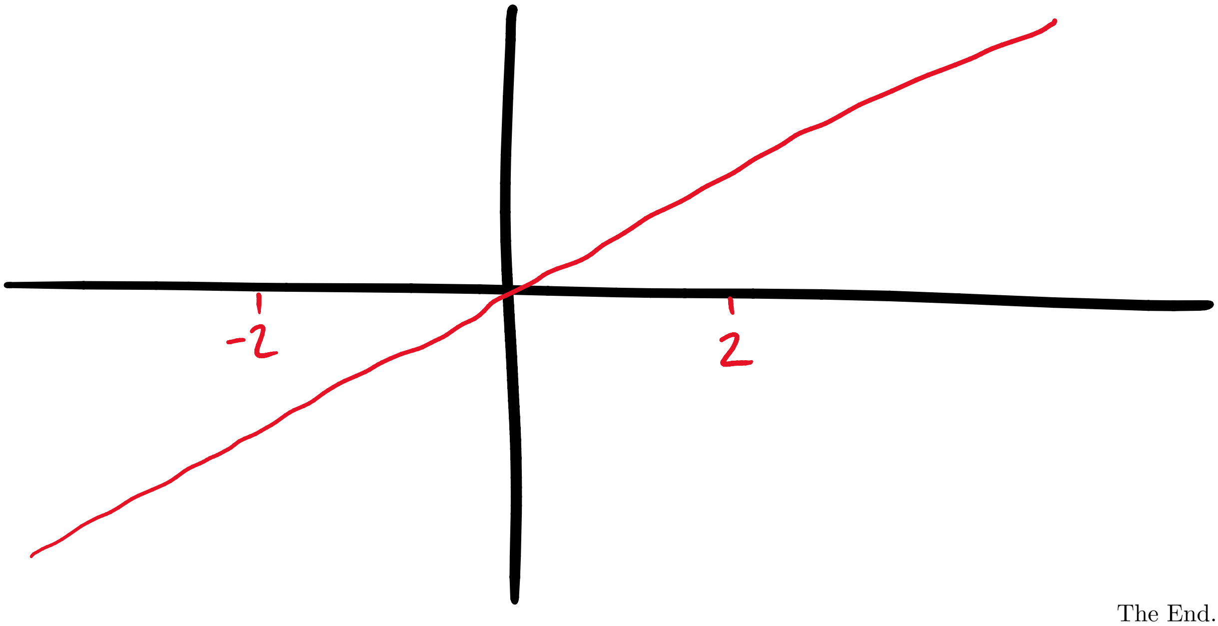
(b) At what intervals is the function $f'(x) < 0$?

Answer: $x > 0$

(c) At what intervals is the function $f'(x) = 0$?

Answer: $x = 0$

(d) Sketch a possible candidate for the function $f'(x)$. You need only label the x-axis.



The End.